Math 201 Quiz 1 Long Sample I N. Nahlus

Name: ……………………………………….. **_I.D** ……………………………………

In computing limits, you may skip the following

1) certain justifications ***if they are well-known.***

2) In LCT, IF you are sure that L=1, ***write L=1 & skip its proof.***

Investigate = Investigate for convergence or divergence

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1. (a) Investigate the series $\sum_{n=1}^{\infty} (\frac{5n+1}{5n+7})^{n}$

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Total over 100	

(b) Investiagte $\sum_{n=1}^{\infty} a_n$ given that $|\frac{a_{n+1}}{a_n}| = \frac{4n^2}{(3n+2)(n+1)} - \frac{\sin(n)}{n}$

(c) Investiagte $\sum_{n=1}^{\infty} a_n$ given that $|\frac{a_{n+1}}{a_n}| = 1 + \frac{n!}{n^n}$

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d) Investiagate $\sum_{n=1}^{\infty} \sin(\frac{5^n}{n!})$

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e) Investigate $\sum_{2}^{\infty} \ln(1 + \frac{3}{n \ln n})$

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f) Investigate $\sum_{n=1}^{\infty} (\sqrt[n]{n} - 1)^2$ (BIG Hint: $\sqrt[n]{n} = e^{\frac{\ln}{n}}$)

2a) Investigate $\sum_{1}^{\infty}(-1)^{n} \frac{100^{n}(n!)^{3}}{(3n)!}$

2a') Find $\lim_{n\to\infty} \frac{n^{3}}{\sqrt[n]{(3n)!}}$ **Hint** : Use $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$ (if this last exists) .

2b) Find the domain of convergence **and the domain of Absolute convergence** of

$$\sum_{n=2}^{\infty} (-1)^n \frac{(2x-3)^n}{7^n \, n \ln n}$$

3a) Find the Maclaurin series of $f(x) = e^{-5x^3}$ to deduce $f^{(150}(0)$

3*b*) ***Estimate*** $\int_0^{0.2} e^{-5x^3}\,dx$ by using the first 3 terms of the appropriate series & find | ***Error*** | < ....

4a) (**Suppose** $\sum_{n=1}^{\infty} a_n$ **converges,** prove or disprove that $\sum_{n=1}^{\infty} \ln(2+\sin(a_n))$ converges

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4b) **Suppose** $\sum_{n=1}^{\infty} a_n$ **converges,** prove or disprove that $\sum_{n=1}^{\infty} (-1)^n \left(\frac{5+\sin(a_n)}{4}\right)^n$ converges

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4c) Investigate carefully $\sum_{n=1}^{\infty} (-1)^n \frac{\sin(n)}{n\sqrt{n}}$

5a) Find $\lim\limits_{n\to\infty} \dfrac{1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\dots\dots\dots+\frac{1}{\sqrt{n}}}{5\sqrt{n}}$

5b) Use Taylor's Remainder's formula to show that $f(x) =$ its Maclaurin series given that $|\, f^{(n)}(x)\,| \le (1000)^{7n}\, e^{x}$ ***for all n*** & ***for all x***.
(For time limitations, assume x > 0)

5B) Use Taylor's Remainder formula to find the maximum possible Error (as a simple fraction)
in the approximation of $\sum_{n=0}^{\infty} \frac{(0.7)^n}{n!}$ using only the first 3 terms.

5c) Use the Maclaurin series of e^x to prove $\sqrt[n]{n!} > \frac{n}{e}$ for all n.